k = film coefficient, min.$^{-1}$
K_d = equilibrium distribution coefficient, $K_d = (q/c)_{\text{equil.}}$, dimensionless
q = solute concentration within resin, g./ml. of internal solution
q_i = initial internal concentration, g./ml.
q_s = internal concentration at surface of resin particle, g./ml.
Q = bulk resin phase concentration, g./ml. of internal solution
r = radial coordinate in resin particle, cm.
R = resin particle radius, cm.
N_{Re} = $\dfrac{D_p\, u\, \rho_{av}}{A\, \epsilon\, \mu_{av}}$ Reynolds number, dimensionless
t = time, min.
T = time required to saturate the column, min.
u = volumetric flow rate, cc./min.
V = total volume of packed bed, cc.
z = axial coordinate of column, cm.

Greek Letters

ϵ = external void fraction, dimensionless
μ_{av} = average solution viscosity, g./(cm.)(sec.)
ρ_{av} = average solution density, g./cc.

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Heat Transfer to Molten Flowing Polymers

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A precise, reproducible method has been developed for measuring temperature profiles in flowing molten polymers with heat transfer. Experimental data determined by this method showed that viscous dissipation occurred, but not at the level predicted theoretically. The difference between the actual and theoretical viscous dissipation was possibly due to flattening of fluid velocity profile, changing physical properties, and fluid viscoelasticity. Nusselt numbers calculated from the data checked theoretical Nusselt-Graetz solutions.

The transfer of heat to molten flowing polymers is an important factor in such polymer processing operations as extrusion and injection molding. The situation is a complex one because molten polymers are compressible and exhibit non-Newtonian and viscoelastic rheological behavior. In addition there is considerable viscous heat generation in such flow systems.

Previous studies of heat transfer to non-Newtonian fluids have generally considered polymer solutions where the effects of viscoelasticity, viscous heat generation, and compressibility are considerably reduced. These studies included the investigations of Winding, Kranich, and Dittman (*1*), Bonilla, Cervi, Colven, and Wang (*2*), Chu, Brown, and Burridge (*3, 4*), Pigford (*5*), Metzner, Vaughan, and Houghton (*6*), Christiansen and Craig (*7*), Petersen and Christiansen (*8*), Metzner and Gluck (*9*), and Oliver and Jensen (*10*).

A number of analytical solutions of the equations of energy and motion applicable to heat transfer to molten polymers have been published. These include the work of Topper (*11*), who considered systems with a heat generation term constant across a tube both for a parabolic velocity profile and potential flow; that of Lyche and Bird (*12*), who studied the Graetz-Nusselt problem for an incompressible power law fluid without heat generation; and various papers by Toor which dealt with the effect of expansion on temperatures with little heat generation (*13*), heat generation and conduction in a viscous compressible fluid (*14*), and heat transfer in forced convection with internal heat generation (*15*). None of the foregoing studies presented any experimental data other than calculated quantities.

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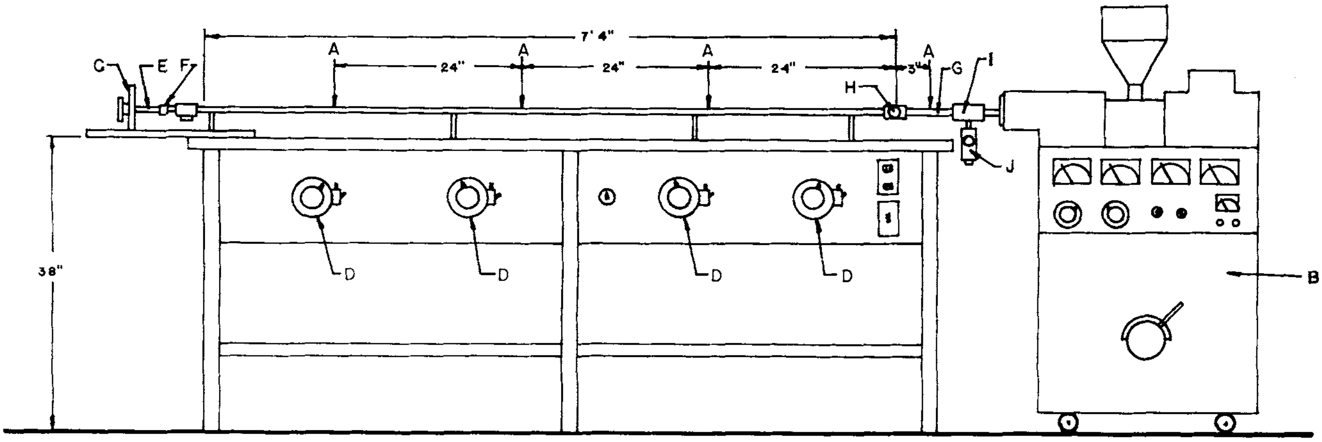

A	SURFACE THERMOCOUPLE	F	TEMPERATURE PROFILE FITTING
B	EXTRUDER	G	MIXING ORFICE
C	TEMPERATURE PROFILE APPARATUS	H	INLET AVERAGE THERMOCOUPLE
D	POWER STATS	I	ADAPTER FITTING
E	TEMPERATURE PROFILE FITTING	J	SAFETY RELIEF VALVE

Fig. 1. Experimental apparatus.

Four studies of heat transfer to molten flowing polymers were found that involved experimental data of one form or another. Two of these, Beyer and Dahl (*16*) and Schott and Kaghan (*17*), dealt mainly with the determination of the radial point at which temperature approximated the mass average temperature of the fluid. The other two investigations, Bird (*18*) and Gee and Lyon (*19*), developed analytical solutions of the equation of motion and energy and then attempted to check these with experimental data.

Bird presented only a very few experimental points, while Gee and Lyon did not present any experimental temperature data. Instead, the latter investigators attempted to justify their analytical solutions on two points, first, that weight flow rates of polymer calculated from their work checked experimental weight flow rates, and second, that their calculated point temperature which had the same value as fluid mass average temperature occurred at the reduced radius found experimentally by Beyer and Dahl (*16*).

The present work was undertaken to determine more extensively experimental temperature profiles during heat transfer to molten flowing polymers and then to use these data to evaluate the analytical solutions developed by other investigators.

An overall schematic of the experimental apparatus is shown in Figure 1. Molten polymer (polyethylene or polypropylene) was pumped by a 1-in. Killion screw extruder (*B*) through an adapter fitting (*I*) to which a safety relief valve (*J*) was attached. The polymer then went through a mixing orifice (*G*) and then to a length of schedule 80, ⅜ in. pipe wrapped with electrical heating tape. Wall temperature was determined by averaging readings from three thermocouples (*A*) embedded in the pipe wall. The determined average deviation between the wall temperature and the individual thermocouple readings was 0.6°F. The apparatus for measurement of fluid point temperatures is designated by (*C*), (*E*), and (*F*).

The method used in this temperature profile measurement system (Figures 2, 3, and 4) was to position the thermocouple parallel to the flow and tube axis so that flow disturbances and heat conduction from the thermocouple tip would be minimized. Both these difficulties were felt to have detracted from the technique used earlier by other investigators (*16, 17*), namely, to insert a thermocouple through a tube wall perpendicular to the flow and then move the device to various radii.

Three and one half inches in length of the thermocouples (Figure 4) were in contact with the flowing polymer. This axial distance was maintained for all experiments.

The points at which temperature measurements were made were from 3 to 18 times the sum of the calculated

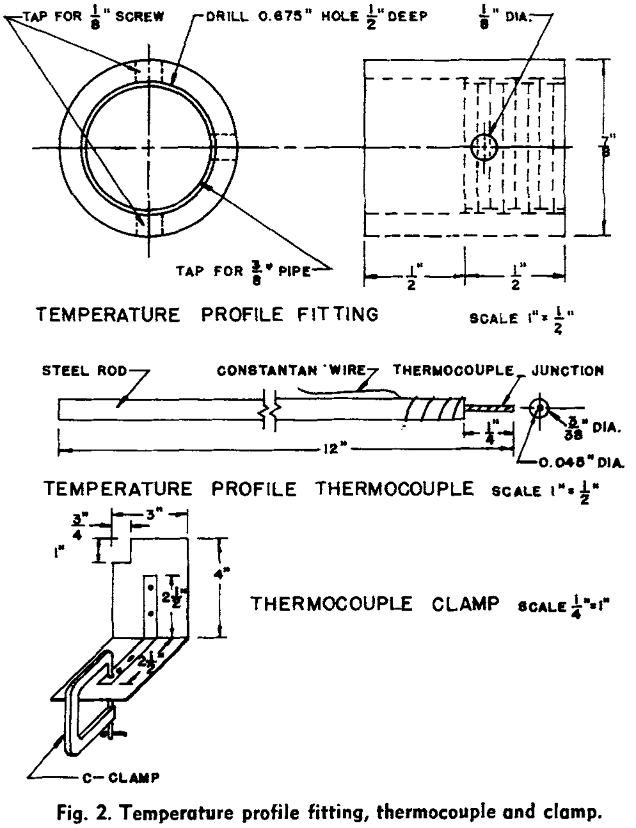

Fig. 2. Temperature profile fitting, thermocouple and clamp.

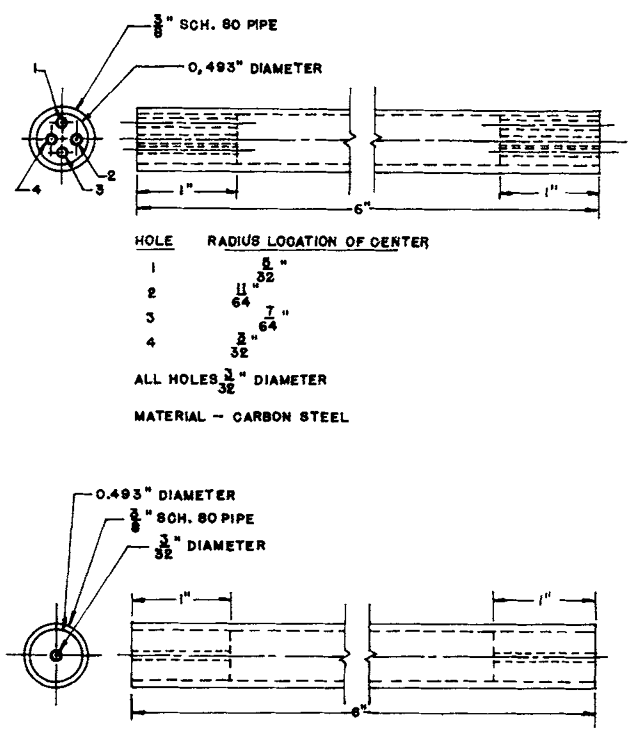

Fig. 3. Temperature profile fittings.

lengths for development of velocity and temperature profiles. These lengths were calculated for the velocity profile development from the work of Bogue (*20*) and for the temperature profile development from the work of Kay (*21*).

EXPERIMENTAL PROCEDURE

The overall apparatus was heated for approximately 2 hr. to melt any polymer remaining from prior experiments. Polymer chips were fed to the extruder whose speed was gradually increased to the desired level. The temperature of the melt entering the test section was then stabilized at 365°F. by manipulating the extruder powerstats and another powerstat which controlled the energy to the melt for the section between the extruder and heat transfer section.

Powerstats for the remainder of the heaters were then adjusted to bring the heat transfer section to a constant temperature. When steady state was attained, as regards no temperature change at a particular radius and angle, readings were taken. These included the polymer inlet temperature, wall temperatures, and the melt temperature at a given radius and angle. Flow rate was also determined by taking a sample of the melt flowing from the output tee for a given time period, cooling the polymer, and then weighing it. Additional readings were taken for every 45 deg. in a complete arc for the particular radius.

When readings were completed for a given radius, the extruder was turned off and the thermocouple shifted to a different radius. Polymer flow was again initiated at the same level and the procedure outlined above was followed. Flow rate was changed after all radii had been checked. The same experimental procedure was again accomplished.

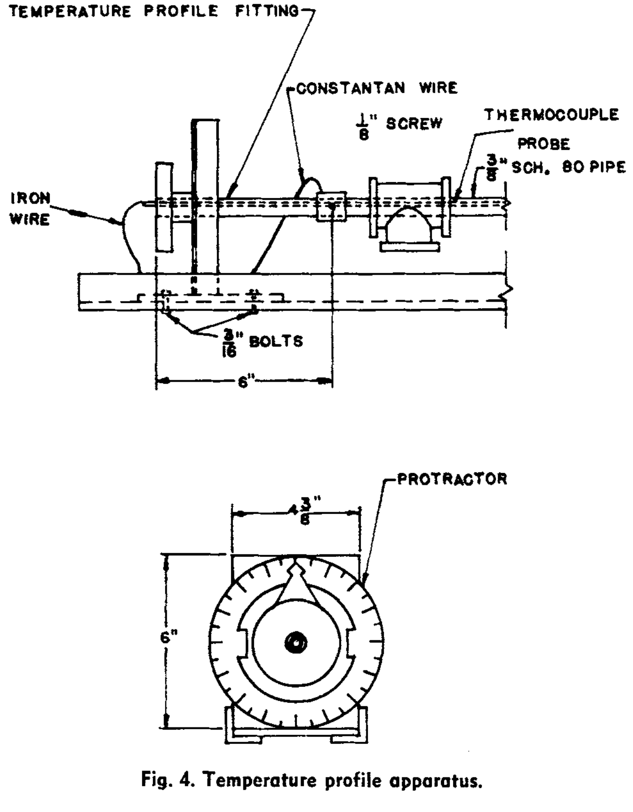

Fig. 4. Temperature profile apparatus.

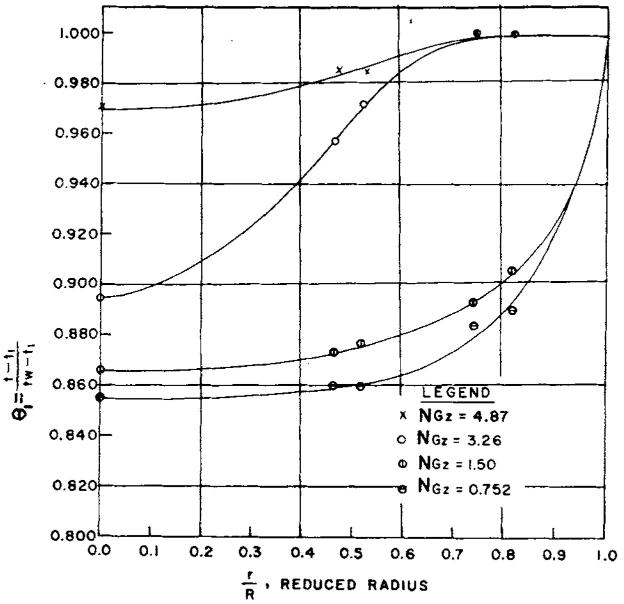

Fig. 5. Experimental dimensionless temperature profiles for polyethylene.

RESULTS

Within the accuracy of the measurement it was found that the average deviation of point temperature was 0.70°F. for all circumferential measurements made at reduced radii of 0.463, 0.519, 0.740, and 0.815. The constancy of temperature circumferentially demonstrated that flow disturbances from the thermocouple, possible effect of thermocouple vibration, and occurrence of natural convection were all negligible.

All the experimental data for polyethylene and polypropylene are plotted as θ_1, a dimensionless temperature, vs. reduced radius in Figures 5 and 6. The polyethylene data are plotted at constant Graetz number. Lack of thermal conductivity data precluded the same treatment for polypropylene.

The only term changing in the Graetz numbers is weight flow rate, since length, specific heat, and thermal conductivity were constant. There are drastic changes in the shape of the curves when Graetz number goes from 1.50 to 3.26 and also from 3.26 to 4.87.

At first glance this behavior seems anomalous, since increasing Graetz number means increasing flow rate and less residence time for the polymer; hence, less opportunity for energy to be transferred to the melt. This reasoning, however, does not consider viscous dissipation in the flowing melt. For a fluid obeying the power law the viscous dissipation factor is

$$\phi_v = K\left(-\frac{\partial V_z}{\partial r}\right)\left|\frac{\partial V_z}{\partial r}\right|^{n+1} \qquad (1)$$

As velocity increases, the gradient will also increase. This is especially pronounced in the region near the wall, since fluid velocity is zero at the wall itself. This accounts for the change in curve shape from a Graetz number of 1.50 to 3.26. The increase in viscous dissipation near the wall is also indicated in Figure 6.

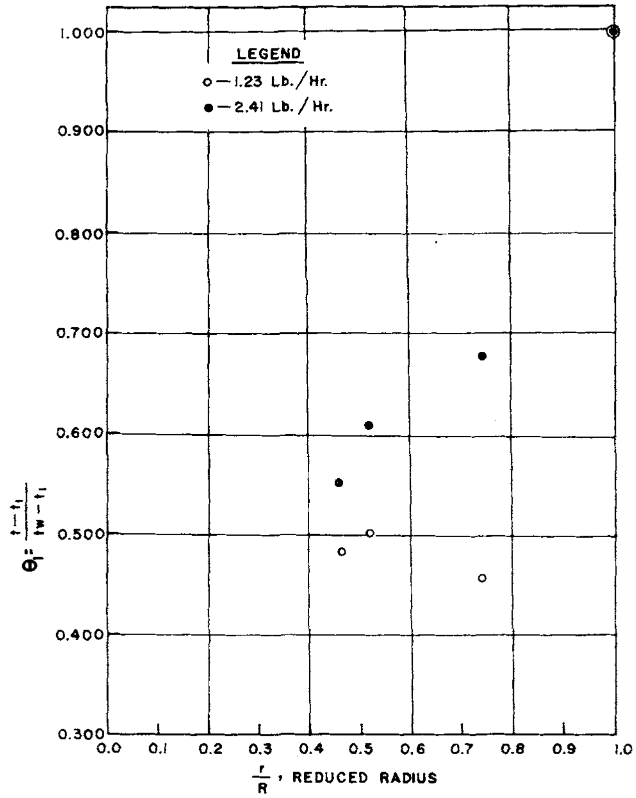

Fig. 6. Experimental dimensionless temperature data for polypropylene.

A possible reason for the change in shape for $N_{Gz} = 3.26$ to $N_{Gz} = 4.87$ is that heat from additional viscous dissipation is transferred to the fluid in the central part of the tube. This would occur since a fairly large temperature gradient exists between the tube center and region near the wall even at $N_{Gz} = 3.26$.

Figures 7 and 8 compare the data of the present work with those calculated from the work of Toor (*14*, *15*). This investigator's work was chosen for comparison since it considered a compressible, non-Newtonian fluid with viscous dissipation. The N_{Br} is a Brinkman number for a power law, non-Newtonian fluid. The more negative the value the greater the viscous dissipation occurring.

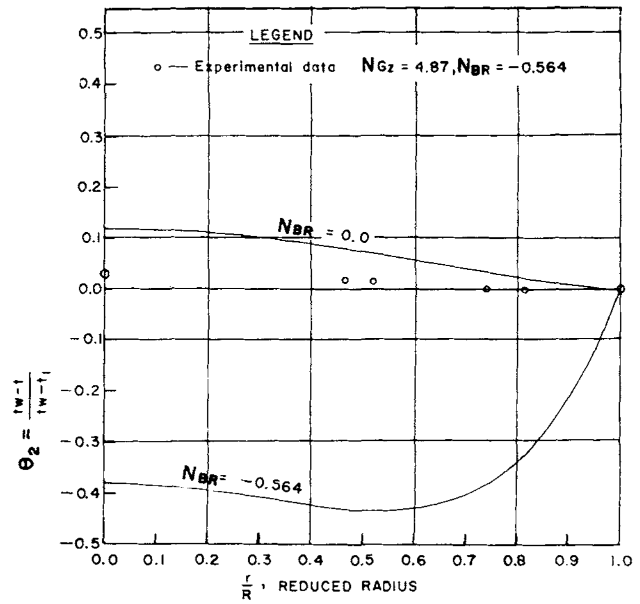

Fig. 7. Comparison of experimental and calculated temperature profiles for $N_{Gz} = 4.87$.

The calculated curves of Figures 7 and 8 are for a non-Newtonian fluid with a flow behavior index of 0.5, whereas the fluid used in this study would have an index of 0.7 under isothermal conditions. Curves for the 0.7 index could not be calculated, since required eigenvalues (*14*, *15*) were not available. The use of a 0.5 index was felt to be reasonable, since the higher wall temperature should flatten the fluid velocity profile and reduce flow behavior index.

As can be seen, the experimental data show some effect of viscous dissipation but not as great an effect as that predicted by Toor.

Several factors could account for this behavior. First, the higher wall temperature would tend to distort the velocity profile in the direction of plug flow where, of course, viscous dissipation is zero. Second, the theoretical approach assumes constant physical properties which is not correct. Finally, there is the viscoelastic nature of the polymer melt. Work done on a purely viscous material is immediately dissipated as heat. However, for a viscoelastic material a portion of the work is not dissipated but stored in the material. In the present study there was

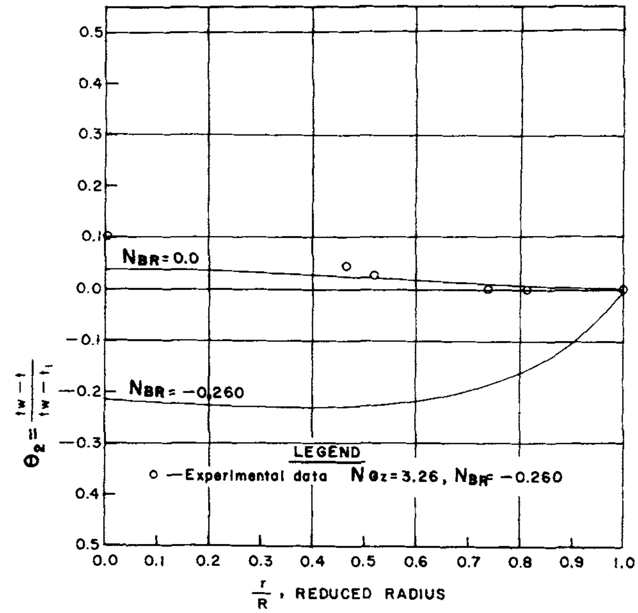

Fig. 8. Comparison of experimental and calculated temperature profiles for $N_{Gz} = 3.26$.

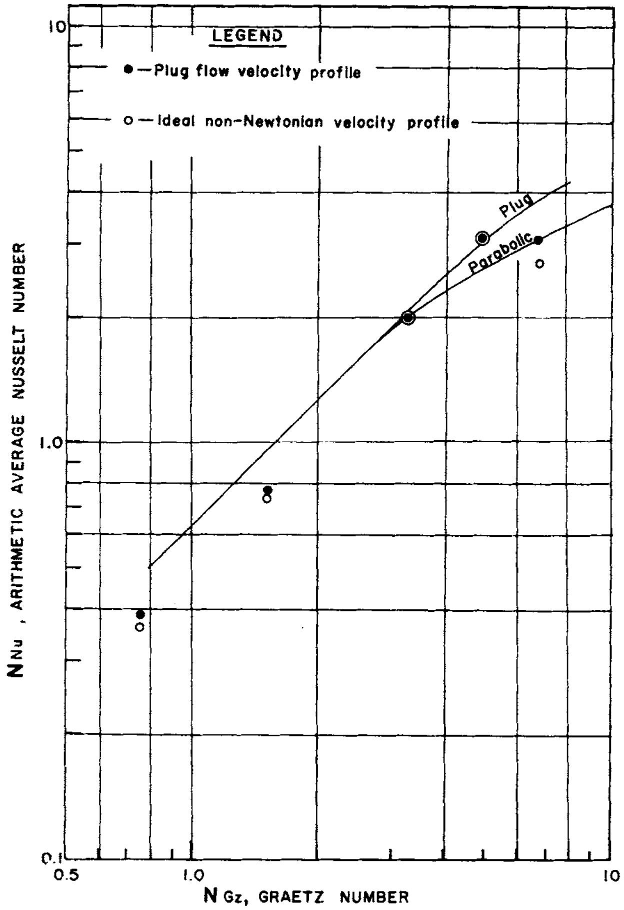

Fig. 9. Comparison of experimental Nusselt-Graetz data with theoretical curves.

considerable evidence of both viscoelasticity and stored energy seen in the bulging of the polymer melt exit streams.

Average Nusselt numbers were also computed from the experimental data. The procedure was as follows. First, two sets of mass average temperatures were calculated from the experimental temperature profiles and two velocity profiles, one for plug flow and the other for a non-Newtonian fluid with a flow behavior index of 0.7. Average values of heat transfer coefficients were then calculated from these mass average temperatures, inlet and wall temperatures, and the heat transfer area. Nusselt numbers were then computed from the heat transfer coefficients.

These calculated Nusselt numbers are compared with the theoretical Nusselt-Graetz curves for plug flow and for a parabolic velocity profile in Figure 9.

The data fall fairly close to the theoretical curves with most of the data below a Graetz number of 2.5 falling below the curve. The agreement is good considering that much of the data reported for Newtonians (*22* to *24*) scatter badly at Graetz numbers less than 10. In some cases the Newtonian data had deviations as high as 400% and generally average deviations were only an order of magnitude less.

CONCLUSIONS

The conclusions reached in the present study are:

1. A method for measuring precise, reproducible temperature profiles in molten flowing polymers has been developed.
2. The effect of viscous dissipation in flow of polymer melts is less than that predicted theoretically.
3. The reasons for the difference in 2 are probably due to viscoelastic effects in the fluid, property changes with temperature, and distortion of fluid velocity profile because of temperature.
4. Nusselt-Graetz data for molten flowing polymers check existing theoretically derived curves.

NOTATION

C_p = specific heat B.t.u./(lb.)(°F.)
h_a = average heat transfer coefficient, B.t.u./(sq. ft.)(°F.)
K = consistency index, (lb.)(hr.^{n})/sq. ft.
k = thermal conductivity, B.t.u./(sq. ft.)(°F.)(ft.)
$N_{Br} = \dfrac{0.0234\,\tau_w\, n}{k\,R\,(3n+1)\,(t_1 - t_w)}$, Brinkman number for power law fluid
N_{Gz} = Graetz number, wC_p/kL
N_{Nu} = Nusselt number, h_aD/k
n = flow behavior index
R = tube outside radius, ft.
r = point radius, ft.
t = point temperature, °F.
t_1 = average fluid inlet temperature, °F.
t_w = wall temperature, °F.
V_z = axial point velocity, ft./sec.
τ_w = wall shear stress, lb.$_f$/sq. ft.
θ_1 = dimensionless temperature, $t - t_1/t_w - t_1$
θ_2 = dimensionless temperature $t_w - t/t_w - t_1$
ϕ = viscous dissipation factor
density, lb./cu. ft.

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